





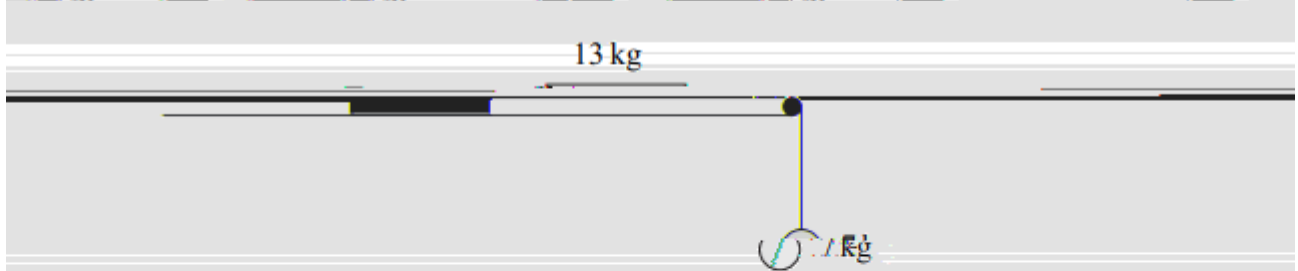
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Year 12 into 13 SIL Practice Paper 1 (101 marks)

1.	<p>Find, to 1 decimal place, the values of <math>\theta</math> in the interval <math>0 &lt; \theta &lt; 180^\circ</math> for which</p> $3\theta + 20^\circ = 1 \cos(3\theta + 20^\circ)$ <p style="text-align: right;"><math>4\sqrt{3} \sin \theta</math></p>
2.	<p>Find in exact form the unit vector in the same direction as <math>\mathbf{a} = 4\mathbf{i} - 7\mathbf{j}</math>.</p> <p style="text-align: right;"><b>(Total 3 marks)</b></p>
3.	<p>Find the exact value of <math>\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)</math>.</p>
4.	<p><math>f(x) = x^3 - 4x^2 - 35x + 20</math>.</p> <p>Find the set of values of <math>x</math> for which <math>f(x)</math> is increasing.</p>
5.	<p>Find the exact value of <math>\int_0^1 x^2 \ln x \, dx</math>.</p>
6.	<p>Find the exact value of <math>\int_0^1 x^2 \ln x \, dx</math>.</p>
7.	<p>Show that <math>\int_2^8 \frac{1}{x} \, dx = \ln 4</math>.</p>
8.	<p>Find</p> <p>(i) <math>\int 8e^{-2x} \, dx</math></p> <p>(ii) <math>\int (4x+5)^9 \, dx</math></p>
9.	<p>Find <math>\frac{dy}{dx}</math> for the following cases:</p> <p>(i) <math>y = \ln(3 + 2x^2)</math></p> <p>(ii) <math>y = \ln(x)</math></p> <p>(iii) <math>y = \frac{x}{2x+1}</math></p>
10.	<p>Find the derivative of <math>y = \frac{x^2 + 4}{x^2 - 4}</math>.</p>



11.	<p>Solve, for <math>0 \leq \theta &lt; 360^\circ</math>, the equation</p> $2 \tan^2 \theta + \sec \theta = 1.$ <p>your answers to 1 decimal place, giving</p> <p style="text-align: right;">(6)</p>
12.	<p>Express <math>\frac{5x+3}{(x-3)(x-2)}</math> in partial fractions.</p> <p style="text-align: center;">(3)</p>
13.	<p>As the term in the expansion of <math>(2x-3)^{10}</math> is <math>2^k x^k (-3)^{10-k}</math>, find the value of <math>k</math> for which the term is a constant.</p>
14.	<p>The diagram shows a block, of mass 13 kg, on a rough horizontal surface. It is attached by a string that passes over a smooth peg to a sphere of mass 7 kg, as shown in the diagram.</p>  <p>the sphere holds the system is released from rest and after 2 seconds the block has a speed of 0.7 m/s and the block has not reached the peg.</p> <p>State two assumptions that you should make about the string in order to model the motion of the sphere and the block. (2 marks)</p> <p>(i) The string is inextensible. (1 mark)</p> <p>(ii) The string is light. (1 mark)</p> <p>(c) Find the tension in the string. (3 marks)</p>



15.

16.



Year 12 into 13 SIL Practice Paper 2 (99 marks)

1.	<p><math>\log_{11} (2x - 1) = 1 - \log_{11}(x + 4)</math>.</p> <p>Find the value of <math>x</math> showing detailed reasoning.</p> <p style="text-align: right;"><b>(Total 6 marks)</b></p>
2.	<p>A particle <math>P</math> of mass <math>6 \text{ kg}</math> is acted on by the action of two forces <math>F_1</math> and <math>F_2</math> which are perpendicular to each other. The forces are given by</p> $F_1 = (8\mathbf{i} - 10\mathbf{j}) \text{ N and } F_2 = (p\mathbf{i} + q\mathbf{j}) \text{ N, } p \text{ and } q \text{ are constants.}$ <p>The acceleration of <math>P</math> is <math>\mathbf{a} = (3\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-2}</math>.</p> <p>(a) Find to 1 decimal place the angle between the acceleration and <math>\mathbf{i}</math>.</p> <p>(b) Find the values of <math>p</math> and <math>q</math>.</p> <p>(c) Find the magnitude of the resultant force <math>R</math> of the two forces <math>F_1</math> and <math>F_2</math>. Simplify your answer fully.</p> <p style="text-align: center;"><b>(3)</b></p>
3.	<p>(a) Sketch the graph of <math>y = 8^x</math> stating the coordinates of any points where the graph crosses the axes.</p> <p>(i) Describe the transformation which transforms the graph <math>y = 8^{x-1}</math> to the graph <math>y = 8^{x-1} + 5</math>.</p> <p>(ii) Describe the transformation which transforms the graph <math>y = 8^{x-1} + 5</math> to the graph <math>y = 8^{x-1} + 5 + 2</math>.</p> <p style="text-align: center;"><b>(1)</b></p>



4.	
5.	
6.	



9.	<p>Solve <math>\tan \theta = 2</math> for <math>0^\circ &lt; \theta &lt; 180^\circ</math> using the equation <math>\tan \theta = 2</math>.</p> $\tan \theta = 2 \Rightarrow \sin \theta = 2 \cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = 2 \Rightarrow \tan \theta = 2$ <p>giving your answers to 1 decimal place.</p> <p>(6)</p>
10.	<p>Find the binomial expansion of <math>(1+x)^{-3}</math> in ascending powers of <math>x</math>. Give each coefficient as a simplified fraction.</p> $(1+x)^{-3} = 1 - 3x + \frac{15}{2}x^2 - \dots$ <p>(5)</p>
11.	<p>Express <math>f(x) = \frac{4-2x}{(x+1)(x+2)(x-3)}</math> in partial fractions.</p> $f(x) = \frac{4-2x}{(x+1)(x+2)(x-3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$ <p>(3)</p> <p>Hence find <math>\int f(x) dx</math>.</p> $\int f(x) dx = \int \left( \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3} \right) dx$ <p>(3)</p>
12.	<p>A block of mass 5 kg is held by a string of mass negligible length at 40° to the horizontal. The string is attached to a wall. The block is released from rest. Find the speed of the block when it is vertically below the point of attachment.</p> <p>(4)</p> <p>In reality, air resistance does act on the block. State how this would change your answer.</p>





13.

A car of mass  $1600\text{ kg}$  tows a trailer of mass  $400\text{ kg}$  on a straight horizontal road. The car starts from rest and accelerates uniformly. The car has a constant resistance of  $100\text{ N}$  and the trailer has a constant resistance of  $200\text{ N}$ .

(a) Find the acceleration of the car.

[3 marks]

(b) Find the tension in the tow bar. The car has a constant resistance of  $100\text{ N}$  and the trailer has a constant resistance of  $200\text{ N}$ .

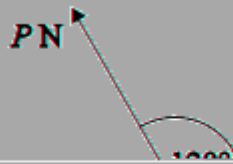
(i) Find the tension in the tow bar.

[3 marks]

(ii) Find  $P$ .

14.

Three forces of magnitude  $40\text{ N}$ ,  $P\text{ N}$  and  $Q\text{ N}$  all act in a horizontal plane. These forces are in equilibrium.  $Q$  is directed horizontally to the right.





Year 12 into 13 SIL Practice Paper 3 (105 marks)

1.	<p>The line with equation <math>mx + ny - 3 = 0</math> touches the circle with equation <math>x^2 + 6x + y^2 - 8y = 4</math>.</p> <p>(a) Find the values of <math>m</math> and <math>n</math> if the line is perpendicular to the radius of the circle at the point of contact.</p>
2.	<p>Given that <math>\vec{a}</math> and <math>\vec{b}</math> are unit vectors and <math>\vec{a} \cdot \vec{b} = \frac{1}{2}</math>, find the angle between <math>\vec{a}</math> and <math>\vec{b}</math>.</p> <p>(a) the vector <math>\overline{AB}</math> in terms of <math>\vec{a}</math> and <math>\vec{b}</math>.</p> <p>(b) Give further that <math> \vec{a}  = 2</math> and <math> \vec{b}  = \sqrt{2}</math>, find the true possible values of <math>\vec{a} \cdot \vec{b}</math>.</p>
3.	<p>A fish tank in the shape of a cuboid is to be made from <math>1600 \text{ cm}^2</math> of glass.</p> <p>(a) Show that the volume, <math>V \text{ cm}^3</math>, of the fish tank is given by <math>V = 400x - \frac{x^3}{4}</math>.</p> <p>(b) Give further that <math>x &gt; 0</math>, use differentiation to find the maximum or minimum value of <math>V</math>.</p>



4.	<p>The line <math>l</math> is defined by the equation <math>3x + 2y = 12</math>. The line <math>m</math> is defined by the equation <math>2x + 3y = 12</math>.</p> <p>The lines <math>l</math> and <math>m</math> intersect at the point <math>A</math>.</p> <p>The line <math>n</math> is defined by the equation <math>2x + y = 6</math>.</p> <p>The line <math>n</math> intersects the line <math>m</math> at the point <math>B</math>.</p> <p>Find the coordinates of the point <math>A</math>.</p> <p>Find the coordinates of the point <math>B</math>.</p> <p>Find the equation of the line through <math>A</math> and <math>B</math>.</p> <p>Find the area of the triangle formed by the lines <math>l</math>, <math>m</math> and <math>n</math>.</p> <p>(Total 9 marks)</p>
5.	<p>Find the value of <math>\int_0^2 \frac{2}{(x-1)^2} dx</math>.</p>
6.	<p>A curve <math>y = \frac{2x+1}{x^2}</math> has the point <math>P(1, 2)</math> on it. Find the equation of the tangent to the curve at <math>P</math>, where <math>a, b</math> and <math>c</math> are integers.</p>
7.	<p>Solve the differential equation <math>\frac{dy}{dx} = 2x + 1</math>.</p>
8.	<p>Find the area of the region bounded by the curve <math>y = x^2</math> and the line <math>y = 2x</math>.</p>
9.	<p>Solve, for <math>0 \leq \theta &lt; 2\pi</math>, the equation <math>3\sec^2\theta + 3\sec\theta = 2</math>.</p> <p>You must show all your working. Give your answers in terms of <math>\pi</math>.</p> <p>(6)</p>



10.	<p> <math>\frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \dots</math> is an ascending series of positive terms.         </p> <p> <math>x + 8</math> </p> <p>           ascending powers of <math>x</math>.         </p>
11.	
12.	<p>           The figure shows the velocity-time graph for a particle moving in a straight line.         </p> <p>           (a) Find the position of the particle at the end of its motion.         </p> <p>           (b) Show that the particle travels a distance of 75 metres during the first 30 seconds of its motion.         </p>



13.

14.



Year 12 into 13 SIL Extension Paper 1 (62 marks)

1	$y = \frac{5x^2 + 10x}{(x+1)^2} \quad x \neq -1$ <p>(a) Show that <math>\frac{dy}{dx} = \frac{A}{(x+1)^n}</math> where <math>A</math> and <math>n</math> are constants to be found. <span style="float: right;">(4)</span></p> <p>(b) Hence deduce the range of values for <math>x</math> for which <math>\frac{dy}{dx} &lt; 0</math>. <span style="float: right;">(1)</span></p>
2	<p>An arithmetic sequence has first term <math>a</math> and common difference <math>d</math>.</p> <p>(a) Show that the sum of the first 10 terms is <math>5a + 45d</math>. <span style="float: right;">(2 marks)</span></p> <p>(b) Show that <math>4a + 70d = 4a^2 + 20ad + 25d^2</math>. <span style="float: right;">(3 marks)</span></p>
3	<p>Three points <math>A</math>, <math>B</math> and <math>C</math> have coordinates <math>A(8, 17)</math>, <math>B(15, 10)</math> and <math>C(7, 2)</math>.</p> <p>(a) Show that the three points <math>A</math>, <math>B</math> and <math>C</math> are collinear. <span style="float: right;">(3 marks)</span></p> <p>(b) <math>A</math>, <math>B</math> and <math>C</math> are on a circle.</p> <p>(b) (i) <math>AC</math> is a diameter of the circle. Explain why. <span style="float: right;">(1 mark)</span></p> <p>(b) (ii) Determine whether the point <math>D(9, 2)</math> lies inside the circle, on the circle or outside. <span style="float: right;">(3 marks)</span></p> <p style="text-align: right;"><b>[4 marks]</b></p>



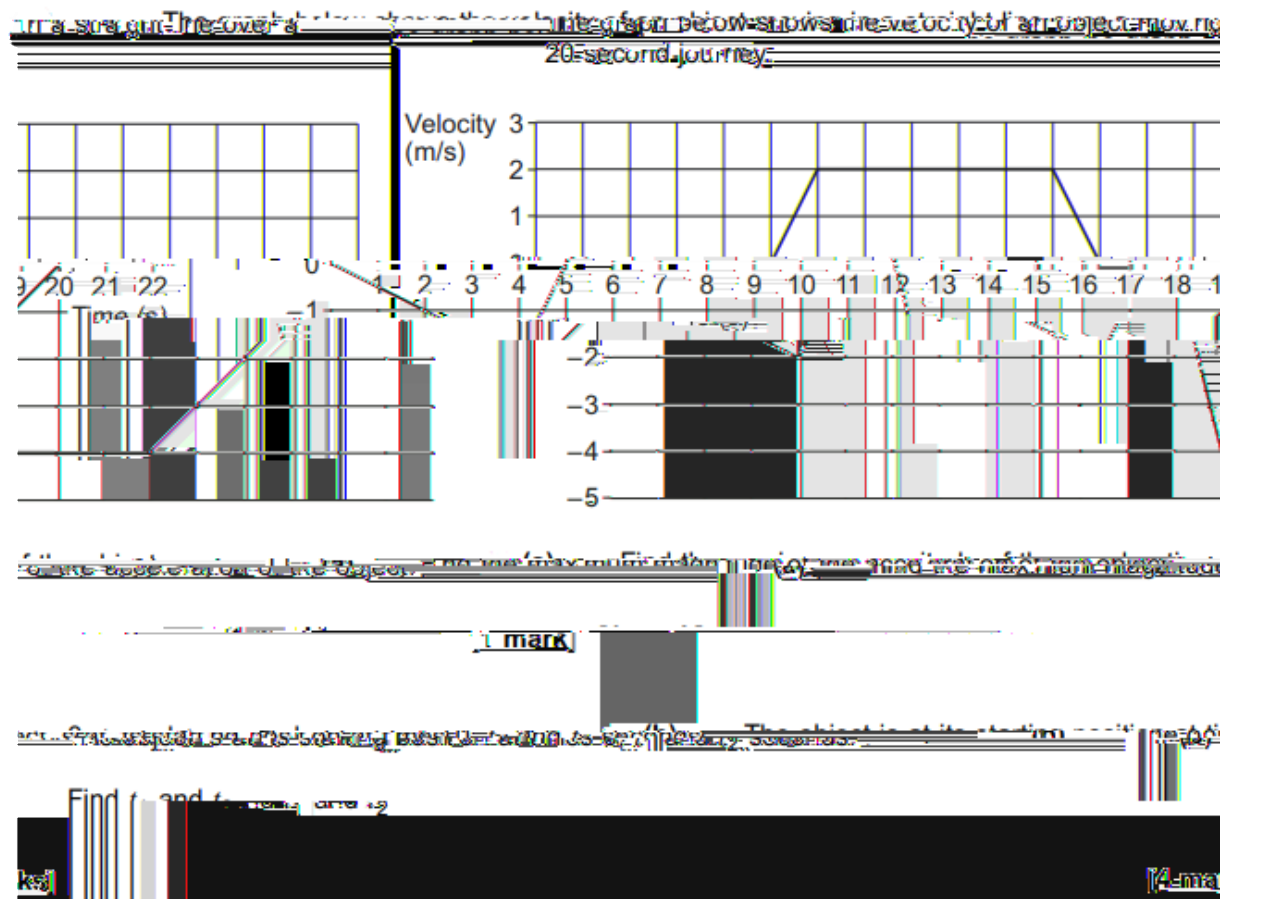


6

The figure shows a graph of  $s$  against  $t$ . The point  $A$  is a stationary point on the curve. The point  $B$  on the curve has  $s$ -coordinate  $4 + k$ .

The result of part (a) can be used to show that  $A$  is a stationary point on (b). Explain how the result of part (a) can be used to show that  $A$  is a stationary point on (b). [2 marks]

7







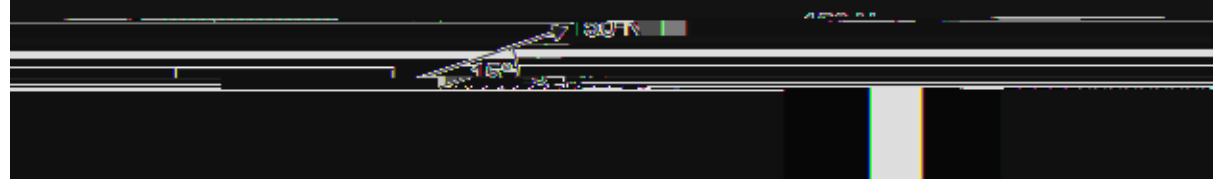
8

In this question use  $g = 9.8 \text{ m s}^{-2}$

ground. The coefficient of friction between the crate and the ground is 0.35.

A boy attempts to move a wooden crate of mass 20 kg along a horizontal surface by applying a force of 450 N at an angle of  $4^\circ$  to the horizontal. Determine whether the crate remains stationary. Fully justify your answer. (5 marks)

A boy attempts to move a wooden crate of mass 20 kg along a horizontal surface by applying a force of 450 N at an angle of  $4^\circ$  to the horizontal. Determine whether the crate remains stationary. Fully justify your answer. (5 marks)



Determine whether the crate remains stationary.

Fully justify your answer.

(5 marks)







A quadrilateral has vertices  $A$ ,  $B$ ,  $C$  and  $D$  with position vectors given by

$$\vec{OA} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}, \quad \vec{OD} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \quad \vec{OB} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad \vec{OC} = \begin{bmatrix} 0 \\ 7 \end{bmatrix},$$

s)

[5 mark

6

A buggy is pulling a roller-skater, as shown in the diagram, by means of a connecting rope as shown in the diagram.



The combined mass of the buggy and unvër is 410 kg

A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg

A total resistance force of  $R$  new

*[Handwritten student work for Question 6, including diagrams and calculations]*

Diagram showing forces on the buggy and roller-skater:

- Buggy:** Driving force  $F = 300\text{ N}$  (right), Resistance  $R = 140\text{ N}$  (left), Weight  $W = 410g$  (down), Normal force  $N$  (up).
- Roller-skater:** Tension  $T$  (right), Resistance  $R$  (left), Weight  $W = 72g$  (down), Normal force  $N$  (up).

Calculations:

For the buggy:

$$F - R = ma$$

$$300 - 140 = 410a$$

$$160 = 410a$$

$$a = \frac{160}{410} = \frac{16}{41}$$

For the roller-skater:

$$T - R = ma$$

$$T - R = 72 \times \frac{16}{41}$$

$$T - R = \frac{1152}{41}$$

$$T = R + \frac{1152}{41}$$